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Finite Element Simulations of Composite Vehicle Structures under Impact Loading

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ABSTRACT

A finite element model for progressive in-plane damage and interfacial delamination was presented for cross-ply composite laminates. The commercial finite element software ABAQUS/STANDARD was used for this study. Each composite lamina was modeled as layers of shell elements based on Mindlin's plate theory. Displacement continuity was established across the layers of shell elements. Interlaminar stresses were recovered at the interface with the use of multi-point constraints. Several material constitutive relations were developed to simulate in-plane damage, such as fiber breakage, matrix cracking and in-plane shear failure. Delamination was simulated by removing the displacement constraints. Cross-ply composite laminates were subjected to impact loading. The results from the finite element analysis compared well with the experimental results.

INTRODUCTION

The high strength-to-weight and high stiffness-to-weight ratios of laminated composites have made them popular in a wide variety of engineering applications, especially in high-performance structures. However, laminated composite structures can take maximum advantage of these properties only when their designs do not overlook the composite weaknesses due to various damage modes. Once the damage modes are understood, a better composite structure design can be achieved.

When used in defense or automotive applications, laminated composites are frequently subjected to impact loading. They are quite susceptible to impact loading and their properties are severely degraded upon impact. Because of their heterogeneity and anisotropy within individual laminae and through the laminate thickness, they undergo complex damage process when subjected to impact loading. Because the damage process is critically important to the success of laminated composites, there is a great need to understand the details of it.

The studies of the damage process of laminated composites subjected to impact loading have been performed both experimentally and computationally. Experimental studies involve preparation of test specimens and impact loading the specimens. Drop weight testing and gas gun testing are usually used for impact investigations. Based on the impact energy and absorbed energy and the inspection of the damaged specimens, the damage process can be studied.

For the computational studies of the damage process, finite element method is frequently used. Computational studies provide a lot of scope for parametric studies leading to a better understanding of the damage process. But, it is very difficult to develop a computational model which is able to capture the damage process and at the same time is computationally efficient. This study attempts to develop such a finite element procedure for the study of the damage process of laminated composites.

The finite element method is an effective and efficient tool for computational studies. Its use in fiber-reinforced laminated composites has been an active area of research for quite some time. Because of the complexity of the damage mechanisms that can occur in laminated composites, the overall mechanical behavior of laminated composites is highly nonlinear and is extremely difficult to model. In order to model the nonlinear response of laminated composites, continuum models are used. Representation of individual composite laminae by a homogenized continuum model permits the overall simulations of lamina performance and the damage process.

A composite laminate can be modeled by using two-dimensional shell elements which use a laminate theory such as the classical laminate theory, Mindlin's plate theory, etc. A composite laminate can also be modeled by using three-dimensional brick elements. Three-

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dimensional elements are able to capture the entire three-dimensional stress distribution (at least theoretically) but they are computationally expensive. Thus, shell elements are the most commonly used elements for the study of composite laminates. A commercial finite element software ABAQUS was used for this study because user defined subroutines could be added to the software with ease. ABAQUS uses shell elements based on the first-order shear deformation theory in which the transverse shear strain is assumed to be constant through the thickness of the shell. A number of shell elements have been presented, based on higher-order shear deformation theories which could calculate the transverse stresses more accurately for the study of laminated composites [1,2].

The damage process in composite laminates is a three-dimensional process. There are intralaminar damage in the form of fiber damage, matrix damage and fiber-matrix debonding and interlaminar damage in the form of delamination.

FAILURE CRITERIA

In the analysis of composite laminates, a wide range of failure criteria have been used. A failure criterion may combine all the possible damage modes into one equation, e.g. Tsai-Hill failure criterion, Tsai-Wu failure criterion, etc. Failure criteria may also consist of a set of equations; each representing a distinct damage mode. The first two major failure criteria were proposed by Hashin [3, 4]; one for fiber damage and the other for matrix damage. The failure criteria were quadratic in nature since a linear criterion usually over-predicted the damage while a polynomial criterion was too complicated. The fiber damage due to tension depends upon the normal stress in the fiber direction and the shear along the fiber. The matrix damage depends upon the normal stress in the matrix direction and the shear stress along the fiber.

Yamada and Sun [5] proposed a failure criterion of a lamina similar to Hashin's fiber failure criterion but used in-situ strengths in the criterion. Chang and Chang [6] proposed phenomological failure criteria which included criterion for matrix cracking, fiber damage and fiber-matrix shear failure. The fiber damage was assumed to be caused by the stress in the fiber direction and the fiber-matrix shear failure was assumed to be due to the normal stress in the fiber direction and the shear stress. Their failure criterion for fiber-matrix shear failure was identical to Hashin's failure criterion for fiber damage while their matrix failure criterion was also similar to Hashin's.

Chang and Lessard [7] proposed another failure criterion accounting for the non-linear nature of shear stress-strain relation in the composite materials. They also used in-situ strengths, similar to Yamada and Sun, in their criteria. Shahid and Chang [8] later on presented a finite element code for the analysis of composite materials based on this failure criterion.

Many other failure criteria were available in the literature. Some of them were based on the failure of isotropic materials and then extended to orthotropic behavior by superimposing several cutoff lines that took into account the experimental results [9] while some used statistical technique to generate failure envelopes [10].

DAMAGE MODELS

Once a failure criterion was met, the material underwent progressive damage. Different damage models were proposed to describe the post-failure behavior of the composite laminates. One of the simplest and most widely used methods for the post-failure modeling was the brittle failure model. Chang and Chang [6] proposed to reduce the elastic constants to zero in order to simulate the post-failure response of composites. This model was called a brittle failure model as the material was assumed to be perfectly brittle and there was a sharp drop in the strength of a damaged composite material. This model was used extensively for the analysis of composite materials [7, 8, 11, 12, 29-34]. It was also used for the damage in the matrix direction in this study.

More recently, progressive failure models with post-failure material softening received a great deal of attention. In general, a macroscopic description of damage could be captured by a constitutive model that exhibited a decrease of stress with increasing strain, i.e. strain softening, beyond the point of failure. This strain softening was based on continuum damage mechanics (CDM) theories [13] in which the net effect of fracture was idealized as a degradation of the elastic moduli of the composite materials. All CDM models provided a mathematical description for the dependence of the elastic moduli on the damage state and the evolution of the damage with respect to the loading and unloading behavior of the composite materials. Typically, CDM models used a simple predefined stiffness reduction function, for example, a linear softening relation, an exponential softening relation, or more complex damage growth laws such as Weibull's function. Bazant and coworkers presented a series of crack band models based on CDM [14-16]. Their models were extensively used for quasi-brittle materials such as fiber composites and concrete.

Johnson et al. [17, 18] proposed a CDM model with linear softening for laminated composites. They tried to correlate the area under the softening zone with the material fracture toughness. Matzenmiller et al. [19] proposed a general constitutive model based on CDM for brittle materials. The damage function could be of any form. They adopted a damage function based on Weibull's statistical function for illustration purpose. Williams et al. [20] implemented the constitutive model proposed by Matzenmiller in commercial finite element code LS-DYNA and studied impact response of laminated composites. They also compared this strain softening model with the brittle failure model proposed by Chang [7]. They concluded that in the post-failure

region, the strain softening model was in a better agreement with the experiments when compared to the brittle failure model.

Schapery and Sicking [21] used a thermodynamically based progressive damage formation and growth model. They used a polynomial evolution law for the damage in the matrix direction. Basu and Waas [22] extended the work done by Schapery and Sicking to include the fiber failure. They assumed that the failure in the fiber is brittle. Upon failure, the stress in the fiber direction dropped to a plateau which was maintained upon further loading.

Bazant [15] showed that strain softening models were not objective and proposed a non-local continuum damage model for the analysis of brittle materials [16]. Kennedy and Nahan [23] implemented a non-local continuum damage model based on exponential softening in LS-DYNA for the analysis of composite materials.

DELAMINATION MODELS

The computational modeling of interlaminar failure, i.e. delamination, was equally challenging. A composite laminate is formed by stacking together individual laminae and these laminae can have different fiber orientations, resulting in non-uniform material properties through the laminate thickness. The mismatch of the material properties between the adjacent laminae could cause the laminae to separate from each other when subjected to loading. This kind of failure was called delamination. Liu [24] proposed a bending stiffness mismatch theory to predict the delamination response of the laminated composites. It stated that the amount of delamination was dependent upon the mismatch of bending stiffness between adjacent laminae. Delamination modeling posed a unique challenge in the composite laminate analysis as it required stress and displacement components in all three dimensions. Delamination modeling could be divided into two groups based on the types of failure criteria being used for the prediction of delamination: strength of materials based approach and fracture mechanics based approach. In both approaches the three-dimensional state of the composite laminate had to be known.

Brewer and Lagace [25] proposed a quadratic criterion based on the interlaminar stresses for delamination prediction. Zhou and Sun [26] formulated the same interlaminar failure criterion utilizing interface stresses. Kim and Soni [27] proposed averaging the interlaminar stresses over an arbitrary critical length of one ply thickness. Fenske and Vizzini [28] proposed a failure criterion which included the in-plane stresses at the interface in the quadratic delamination criterion. Three-dimensional brick elements were used in order to calculate the interlaminar stresses. De Moura and Maques [29] used the interlaminar stresses calculated by a shear flexible shell element to predict the delamination in composite laminates subjected to impact loading. However, their study was limited to predicting the delamination shape and not to simulate delamination.

Lou et al. [30] used three-dimensional brick elements and used Chang brittle failure criteria to study impact response of laminated composites. The quadratic failure criterion was used for delamination. Once the criterion was met, the element's stress bearing capacity in the out-of-plane direction was reduced to zero. Other researchers have also used a similar finite element model to predict the impact response of laminated composites [31-34]. These models were computationally expensive owing to the use of three-dimensional brick elements. Moreover, a very refined mesh was required in order to calculate the inlerlaminar stresses accurately when using a brick element.

Li et al. [35] proposed an interface element which joined the brick elements on either side of the potential delamination interface. Once the delamination took place, the stiffness of the interface element was changed to make the displacements discontinuous across the interface. Sprenger et al. [36] used a similar interface element to simulate delamination.

Because of high computational cost involved in using brick elements, focus was shifted to model delamination by using computationally efficient shell elements. Multi-layered shell elements were used to model the composite laminates. Composite laminae on either side of the potential delamination surface were modeled by shell elements. Displacement continuity conditions were enforced between the shell elements.

Zheng and Sun [37] imposed constraints between appropriate nodes of the shell elements to enforce displacement continuity. The strain energy release rate was calculated at the interface by the virtual crack closure method and the constrained conditions between the nodes were removed when the strain energy release rate was higher than the critical value. Klug and Sun [38] also used this model for the study of delamination. Wisnom and Chang [39] developed three-dimensional spring element to enforce the displacement continuity between the corresponding nodes. Their study, however, was based on plain strain elements. The area under the force-deflection curve of the spring was used to calculate the strain energy release rate. Once the delamination criterion was met, the stiffness of the spring was reduced to simulate displacement discontinuity across the interface.

Other researchers used specially developed interface elements to model delamination [40-46]. Interface elements satisfy the displacement continuity conditions in a penalty sense, i.e., the stiffness of the interface element represented the penalty parameter. In contrast to continuum elements where the stress-strain relations were used, the relations between stresses and associated displacements of the interface governed the behavior of the interface elements. With the use of force

and displacement components across the interface, the strain energy release rate across the interface could be calculated and the stiffness of the interface element was reduced to reflect the displacement discontinuity. With the use of interface element, the finite element modeling (preprocessing) was more convenient than the constrained conditions for the displacement continuity. Moreover, with the interface element, the continuity conditions were removed gradually, by changing the stiffness (penalty parameter) of the interface element gradually, resulting in a better representation of the damage process.

IN-PLANE FAILURE

A constitutive model is a phenomenological description of material behavior. In establishing a constitutive model, it is necessary to characterize the behavior of the composite material of interest by testing methods. Once the mechanical responses and damage modes of the composite material are identified, mathematical formulations can be established or selected to simulate the composite behavior. The discussions henceforth describe the composite responses and the incorporation of damage modes into a constitutive model.

BASIC ASSUMPTIONS

The following assumptions were made in developing a constitutive model for a composite lamina.

1. A composite lamina was assumed to be homogeneous. Its elastic constants were averaged properties obtained from material tests.

2. Each composite lamina was modeled by a two-dimensional orthotropic continuum. The orthotropic nature of the composite material was maintained throughout the damage process.

3. Although the stress-strain curves for a composite lamina showed some degrees of nonlinearity, especially in shear, linear elastic relations were assumed till the point of failure.

4. The damage process of a composite lamina was considered to be linearly brittle. This was due to the fact that very small permanent deformations remained in the composite lamina after unloading, as long as the lamina was not very close to complete failure.

STIFFNESS DEGRADATION

As mentioned above, a two-dimensional linear elastic orthotropic model was assumed to represent the behavior of a composite lamina till the point of failure. From Hooke's law, the following stress-strain relation was given

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_{11}}{1-\nu_{12}\nu_{21}} & \frac{E_{22}\nu_{12}}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{E_{11}\nu_{21}}{1-\nu_{12}\nu_{21}} & \frac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} \tag{1}$$

where σ is stress, ε and γ are strains, E is Young's modulus, ν is Poisson's ratio, G is shear modulus and the subscripts 1 and 2 represent the directions of fiber and matrix, respectively. When a composite lamina undergoes damage, the damage may manifest itself in different forms. They can be divided into fiber damage and matrix damage.

FIBER DAMAGE

The stress in the fiber direction of a composite lamina is predominantly carried by the fibers because of their high stiffness in comparison with matrix material. Thus, the stress bearing capability in the fiber direction is not significantly affected by the damage in matrix.

Fibers may fail because of tensile breakage. Broken fibers may debond from matrix and cause cavities. Fibers may also buckle or kink due to compressive loading. These fiber failures are catastrophic and can strongly affect the stress bearing capability of the composite lamina to which the fibers belong. The following failure criteria proposed by Hashin [3-4] were used to predict the fiber failure in this study.

1. Fiber Failure in Tension: $\sigma_{11} \geq 0$

$$e_f^{\,2} = \left(\frac{\sigma_{11}}{X_t}\right)^2 - 1 \quad \begin{cases} \geq 0 \ \textit{failed} \\ < 0 \ \textit{elastic} \end{cases} \tag{2}$$

2. Fiber Failure in Compression: $\sigma_{11} < 0$

$$e_f^{\,2} = \left(\frac{\sigma_{11}}{X_c}\right)^2 - 1 \quad \begin{cases} \geq 0 \ \textit{failed} \\ < 0 \ \textit{elastic} \end{cases} \tag{3}$$

where X_t and X_c are lamina strengths of the composite lamina along the fiber direction in tension and in compression, respectively. Fiber damage could also affect the integrity of matrix and hence all stiffness components in the constitutive model should be degraded when fiber damage takes place. As an example, Fig. 1 shows a uniaxial loading-unloading-reloading curve. As can be seen, it is an elastic-brittle response with a linear stress-strain relation till the point of failure. At the point of failure, the stress value drops to a low level and the composite lamina continues to

maintain the same stress level on further loading. If unloading takes place, the material unloads with a secant modulus. When the stress level becomes zero, there is no permanent deformation in the composite lamina.

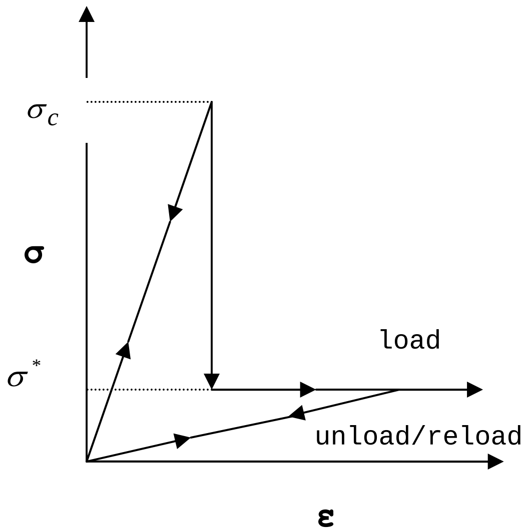


Fig. 1 - Uniaxial stress-strain curve.

When a composite lamina is damaged, the stiffness matrix given in Eq. (1) took the following form:

$$\begin{bmatrix} \frac{E_{11}^*}{1-\nu_{12}^*\nu_{21}^*} & \frac{E_{22}^*\nu_{12}^*}{1-\nu_{12}^*\nu_{21}^*} & 0 \\ \frac{E_{11}^*\nu_{21}^*}{1-\nu_{12}^*\nu_{21}^*} & \frac{E_{22}^*}{1-\nu_{12}^*\nu_{21}^*} & 0 \\ 0 & 0 & G_{12}^* \end{bmatrix} \quad (4)$$

where the quantities in * represent the corresponding degraded elastic constants. They were defined as follows.

$$E_{11}^* = \sigma_{11}^* / \varepsilon_{11}$$

$$\nu_{12}^* = (\nu_{12} / E_{11}) E_{11}^*$$

$$E_{22}^* = E_{22} /1000 \quad (5)$$

$$\nu_{21}^* = \nu_{12}^* E_{22}^* / E_{11}^*$$

$$G_{12}^* = G_{12} /1000$$

It should be noted that

$$\sigma_{11}^* = \sigma_{11}^c /5$$

where σ_{11}^c is the stress to cause fiber failure.

The degradation factor 1/1000 was chosen for the property degradation because if the property was degraded further, it gave convergence problems, while a degradation factor greater than 1/1000 would result in large post failure stress at large strains. For the degradation of E_{11}, a degradation factor 1/5 was chosen as it gave good match with the experimental results. Degradation factors of 1/3, 1/10 and 1/100 were also investigated. The degradation factors 1/3 and 1/10 gave almost similar results as the factor 1/5 but for the degradation factor of 1/100, the damage was too severe and the impactor penetrated the laminate at much lesser energy than in the experiments.

MATRIX DAMAGE

Shear stress and transverse normal stress can transfer to both fibers and matrix. But their effects on composite damage are predominantly restricted to matrix damage and fiber-matrix debonding. In a composite lamina, an initial crack in the matrix can propagate in any direction. However, when the crack reaches a fiber-matrix interface, it changes its course to propagate along the fiber direction without crossing into the fiber. Thus, the damage in matrix is aligned in the fiber direction. This damage of matrix depends upon the matrix strength in both normal direction and shearing direction, if debonding between fiber and matrix is not of concern.

Being different from matrix cracking that is observed due to transverse tensile stress and shear stress, matrix crushing is observed due to transverse compressive stress. A number of failure criteria were proposed for matrix damage. In this study, failure criteria proposed by Hashin and modified by Chang and Lessard were used. It contained the following basic items.

1. Matrix Cracking due to Shear Stress and Transverse Tensile Stress: $\sigma_{22} \geq 0$

$$e_m^2 = \left(\frac{\sigma_{22}}{Y_t}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 - 1 \quad \begin{cases} \geq 0 \ failed \\ < 0 \ elastic \end{cases} \quad (6)$$

where Y_t and S are the transverse tensile strength and shear strength of the composite lamina, respectively. The elastic constants were degraded to 0.1% of their undamaged values to simulate the brittle failure. The damaged stiffness matrix was defined as

$$\begin{bmatrix} \frac{E_{11}}{1-\nu_{12}\nu_{21}^*} & \frac{E_{22}^*\nu_{12}}{1-\nu_{12}\nu_{21}^*} & 0 \\ \frac{E_{11}\nu_{21}^*}{1-\nu_{12}\nu_{21}^*} & \frac{E_{22}^*}{1-\nu_{12}\nu_{21}^*} & 0 \\ 0 & 0 & G_{12}^* \end{bmatrix} \quad (7)$$

where $E_{22}^* = E_{22}/1000$

$$\nu_{12}^* = (\nu_{12} / E_{11})E_{11}^*$$

and $G_{12}^* = G_{12}/1000$

2. Matrix crushing due to transverse compressive stress: $\sigma_{22}<0$

$$e_m{}^2 = (\frac{\sigma_{22}}{Y_c})^2 + (\frac{\sigma_{12}}{S})^2 - 1 \quad \begin{cases} \geq 0 \; failed \\ < 0 \; elastic \end{cases} \quad (8)$$

where Y_c is the transverse compressive strength of the composite lamina. The stiffness matrix for the damaged composite was modified as

$$\begin{bmatrix} \frac{E_{11}}{1-\nu_{12}\nu_{21}^*} & \frac{E_{22}^*\nu_{12}}{1-\nu_{12}\nu_{21}^*} & 0 \\ \frac{E_{11}\nu_{21}^*}{1-\nu_{12}\nu_{21}^*} & \frac{E_{22}^*}{1-\nu_{12}\nu_{21}^*} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (9)$$

Both E_{22}^* and ν_{21}^* are defined in the same manner as in matrix cracking, Eqn. (7). The constitutive model consisting of the failure criteria and degraded stiffness matrices given above was incorporated into the commercial finite element code ABAQUS by programming it into a user's material subroutine UMAT for the analysis of the composite material. As nonlinear equations are solved in increments, the solution process was incremental. For the n^{th} increment, the strain increment for the n^{th} increment and the stresses and strains of the $(n-1)^{th}$ increment were passed on to the subroutine by the software. The solution of nonlinear equations requires the tangent stiffness matrix which depends upon the material constitutive relations. Thus in the subroutine UMAT, the material constitutive relations to calculate the stresses at the end of the n^{th} increment and the tangent stiffness matrix for the n^{th} increment were defined.

DELAMINATION FAILURE

Composite laminates are heterogeneous and anisotropic. Their properties vary from one constituent to another within one lamina and change from one lamina to another through the laminate thickness. Because of the mismatch in material properties, the nonuniform stress distribution through the laminate thickness can cause delamination in composite laminates. As an example, a [0/90] laminate has the highest mismatch of material properties among the [0/θ] laminates, and is the one that is most susceptible to delamination. Delamination is a major form of damage as it severely degrades the properties of a composite laminate and renders the composite laminate prone to further damage. In fact, once delamination takes place in a composite laminate subjected to impact loading, it is very easy for the impactor to run through the composite laminate by cracking the matrix and pushing away the fibers in individual laminae. Thus, delamination is an important characteristic of composite damage and composite simulation should include delamination modeling.

DELAMINATION MODELING AND CRITERIA

Two delamination models were commonly used, namely the multi-point constraint model and the interfacial layer model. In principle, both were based on using multi-point constraints to enforce the continuity of displacement across the laminate interfaces and to calculate interlaminar stresses using the constraint forces. Hence, both models gave similar delamination simulations. However, the two models differed from each other on how the continuity of displacements across the laminate interfaces was removed when the criterion for delamination was met. In the multi-point constraint model, the both rotational and translational displacement continuity conditions at the nodes were removed in order to allow displacement discontinuity. In the interfacial layer model, the stiffness of the interfacial element was reduced in order to introduce the displacements discontinuity. Since an additional shell element layer was used as the interfacial layer, the interfacial layer model was computationally more expensive than the multi-point constraint model. Thus, the multi-point constraint model was used in this study.

A number of delamination models can be found in the literature. These criteria can be divided into two groups, the fracture-mechanics based models and the strength-of-materials based models. In the fracture-mechanics based models, an initial delamination front must be assumed along with a delamination failure criterion to calculate the onset of delamination. The propagation of delamination front, for example, can be determined by the strain energy release rate obtained from the virtual

crack closure method i.e., if the strain energy release rate exceeds a critical level, the delamination propagation will take place. In the strength-of-materials based failure models, it is assumed that delamination takes place due to the presence of large interlaminar stresses. Failure criterion based on interlaminar stresses and interlaminar strengths are used to predict the delamination onset. In this study, strength-of-materials based models were used.

To begin with, a quadratic failure criterion was used to predict the onset of delamination. This criterion has the following form:

For $\sigma_{33} \geq 0$:

$$e_d^2 = (\frac{\sigma_{13}}{S})^2 + (\frac{\sigma_{23}}{S})^2 + (\frac{\sigma_{33}}{Y_t})^2 - 1 \quad (10)$$
$$\begin{cases} \geq 0 \text{ delamination} \\ < 0 \text{ no delamination} \end{cases}$$

where S is the interlaminar shear strength and Y_t is the interlaminar transverse strength in tension. It is assumed that delamination takes place only if the interlaminar normal stress σ_{33} is in tension. This criterion was used by many researchers [25-34] and was also used in this thesis research for simulations of composite laminates subjected to impact loading.

Delamination Failure Only

A 125 mm x 125 mm composite laminate with a stacking sequence of [0/90/0] and thickness of 3.81 mm was impacted with an impactor of 12.88 kg mass at a velocity of 3.0 m/s. The composite laminate was modeled by two shell elements. The top shell element represented the 0/90 laminae while the bottom shell element the bottom 0 lamina. Thus, delamination was simulated to take place on the interface between the 0/90 lamina and the bottom 0 lamina. The use of a single interface in delamination simulations is due to the limitation of the commercial finite element code used in this study. For a laminate with two interfaces, three shell elements will have to be used, i.e. each shell element represents a lamina. However, the process of assembling the three laminae to become a composite laminate will render all the nodes but one in the laminae to become dependent variables. The nodes of the top lamina must be independent for the contact algorithm to work in ABAQUS Standard. Thus, the displacement continuity conditions are applied to the corresponding nodes of the middle lamina and makes the degrees of freedom of the nodes of the middle lamina dependent upon the nodes of the top lamina. Hence, the nodes of the middle lamina are unavailable for imposing the displacement continuity conditions between the middle and the bottom laminae. The nodes of the top lamina will have to be used to enforce the continuity conditions. Accordingly the nodal forces in the bottom lamina will have the combined effect of the top and the middle laminae and thus, cannot be used to calculate the interlaminar stresses at the bottom interface. Besides, it has been noted that the bottom interface has the major delamination damage when subjected to impact loading. It was with these reasons, only two shell elements were used to model the three-lamina composite laminate.

When the delamination failure criterion, Eq. (10), was used for modeling composite laminates without an in-plane failure criterion, it predicted delamination shape well, as shown in Fig. 2. The black elements in the center of the composite laminate shown in Fig. 2(a), collectively represent the delamination area. They were identified from the delaminated nodes shown in Fig. 2 (b), to which they attached. As can be observed from Fig. 2, the typical peanut-shaped delamination is observed with the major axis aligned in the fiber direction of the bottom 0^0 lamina.

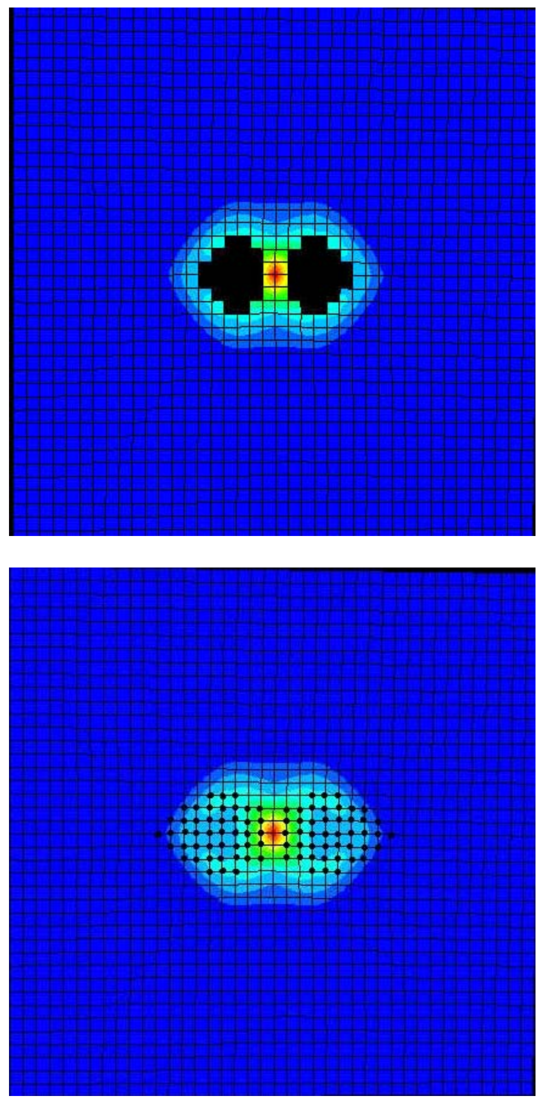


Fig. 2 - Delamination at the bottom interface of [0/90/0] laminate with no in-plane failure.

In-Plane Failure and Delamination Failure

When the delamination failure criterion was used in conjunction with an in-plane failure criterion, it was observed that the delamination size was greatly reduced as shown in Fig. 4.2. This was contrary to what was expected. The in-plane damage modes of the composite material such as matrix damage should have led to increased delamination. In fact, it was observed that the interlaminar shear stresses decreased and the interlaminar normal stress increased when the material stiffness was degraded upon matrix failure. The combination of the changes of interlaminar stresses resulted in the reduction of delamination area. Finite element simulations revealed that the delamination area based on the delamination failure criterion and an in-plane failure criterion proposed in section 3.43 was found to be only 2.73 cm^2 while that based on the delamination failure criterion only was 4.29 cm^2.

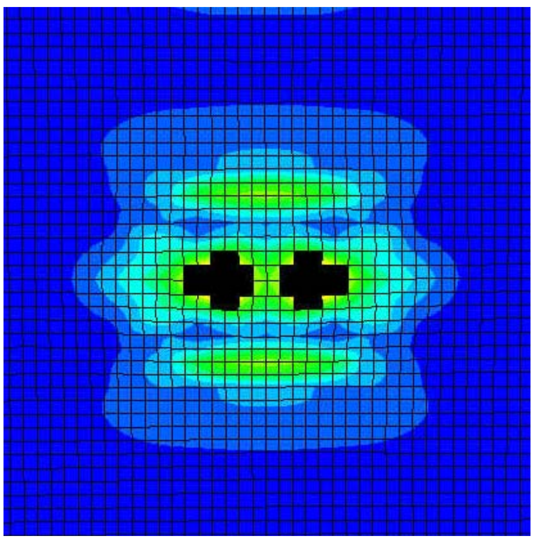


Fig. 3 - Delamination at the bottom interface of [0/90/0] laminate with in-plane failure criterion.

Modified Delamination Failure

It became clear that the foregoing delamination failure criterion must be modified. Fenske and Vizzini [28] proposed a delamination failure criterion with in-plane stress components. They claimed that delamination usually took place in thin matrix rich layers on laminate interfaces. Hence, all six stress components should be included in the justification of delamination onset. The foregoing studies seemed to support their theory. Thus, the delamination failure criterion was modified to include the in-plane stresses in this thesis research.

The original delamination criterion proposed by Fenske and Vizzini takes the following form:

$$e_d^2 = \frac{\sigma_I^2 - \sigma_I\sigma_{II} + \sigma_{II}^2}{Y_t Y_c} + \frac{(Y_c - Y_t)(\sigma_I + \sigma_{II})}{Y_t Y_c} + (\frac{\sigma_{13}}{S})^2 + (\frac{\sigma_{23}}{S})^2 + \left(\frac{\sigma_{33}^2}{Z_t Z_c} + \frac{(Z_c - Z_t)\sigma_{33}}{Z_t Z_c}\right) - 1 \begin{cases} \geq 0 \text{ delamination} \\ < 0 \text{ no delamination} \end{cases} \tag{11}$$

where σ_I and σ_{II} are in-plane principal stresses, Y_t and Y_c are matrix strengths in tension and compression, respectively, S is interlaminar shear strength and Z_t and Z_c are interlaminar normal strengths in tension and in compression, respectively.

It should be noted that both in-plane stresses and interlaminar stresses are included in the delamination failure criterion. However, Eq. (11), the distinction between the tensile interlaminar normal stress and the compressive interlaminar normal stress is not very strong. Although it is well known that the compressive interlaminar normal stress can act against delamination. The fact that delamination has a peanut shape consisting of two discontinuous lobes upon impact seems to support this argument. Hence, the two delamination lobes may not be discontinuous at the point of impact if the delamination failure criterion does not distinguish between interlaminar normal compression and interlaminar normal tension sufficiently.

The Fenske-Vizzini delaminaiton failure criterion is modified to enhance the role of the compressive interlaminar normal stress. It is proposed that delamination takes place only when the interlaminar normal stress is in tension. Thus, the delamination criterion takes the following form:

For $\sigma_{33} \geq 0$:

$$e_d^2 = \frac{\sigma_I^2 - \sigma_I\sigma_{II} + \sigma_{II}^2}{Y_t Y_c} + \frac{(Y_c - Y_t)(\sigma_I + \sigma_{II})}{Y_t Y_c} + (\frac{\sigma_{13}}{S})^2 + (\frac{\sigma_{23}}{S})^2 + (\frac{\sigma_{33}^t}{Y_t})^2 - 1 \begin{cases} \geq 0 \text{ delamination} \\ < 0 \text{ no delamination} \end{cases} \tag{12}$$

It may be noted that Eq. (12) is an extension of Eq. (10), with additional terms accounting for in-plane stresses. To calculate the in-plane stress on the laminate interface with the multi-point constraint model, an additional layer of matrix must be introduced in the finite element analysis. Thus, the [0/90/0] laminate which was modeled by two shell elements, the top element representing the 0/90 lamina while the bottom element the bottom 0^o lamina, is remodeled as [0/90/M/0]. The top shell element represented the 0/90/M lamina and the bottom element the bottom 0 lamina. The additional matrix layer

was very thin with a thickness of 0.01mm and did not have a significant effect on the global response of the composite laminate. With this additional matrix interfacial layer, it was possible to obtain the in-plane stresses on the laminate interface for delamination analysis.

When the impact simulation similar to the previous cases was performed with the modified delamination failure criterion, a delamination area of 7.03 cm^2 was obtained.

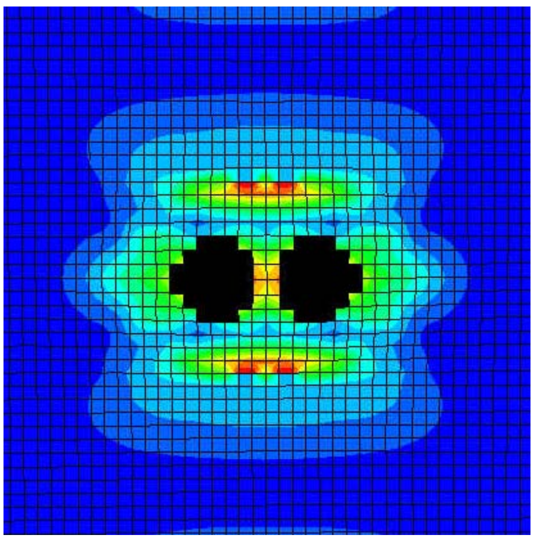


Fig. 4 - Delamination at the bottom interface of [0/90/0] laminate based on modified delamination failure criterion.

SIMULATION OF [0/90/0] LAMINATES

Having proposed a constitutive model to express in-plane damage (including fiber failure and matrix damage) and out-of-plane damage (such as delamination damage), composite laminates with a stacking sequence of [0/90/0] were studied. Because of the large mismatch in material properties through the laminate thickness, delamination was expected to be severe for the cross ply laminate. Once again, composite laminates with dimensions of 125 mm x 125 mm and thickness of 3.81 mm were investigated. Each of them was completely constrained at all four edges and was impacted with a spherical impactor of 12.7 mm diameter and 12.88 kg mass at different impact velocities.

For the finite element simulations, a two-shell-element model was used. As the delamination model could handle only one interface, the delamination on the top interface of the 0/90 lamina was neglected, i.e. the 0/90 laminae were modeled by one shell element. The second shell element modeled the bottom 0 lamina. The composite constitutive model of progressive fiber damage and brittle matrix damage presented in a previous section was used to model the in-plane damage of the composite laminates. The delamination modeling presented earlier was used for modeling the out-of-plane damage. The impactor was modeled as an analytical rigid body and the shell element S4R of ABAQUS was used for the composite lamianae.

Fig. 5 shows the force-deflection curves from the impact simulations for different impact velocities. There were some convergence problems in the simulations at high impact velocities. The simulations stopped near the maximum load. At High impact velocities, the simulations were unable to capture the penetration process. Figs. 6(a), 6(b) and 6(c) compare the load-deflection curves from those from the experiments with the simulations.

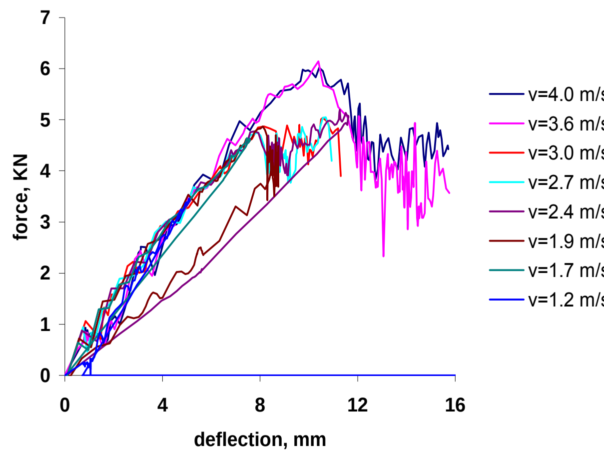
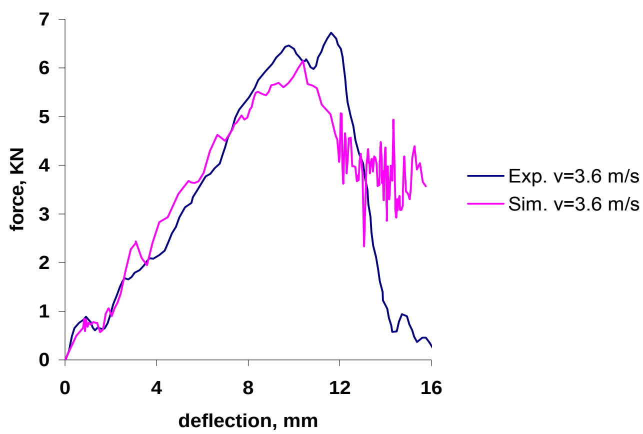
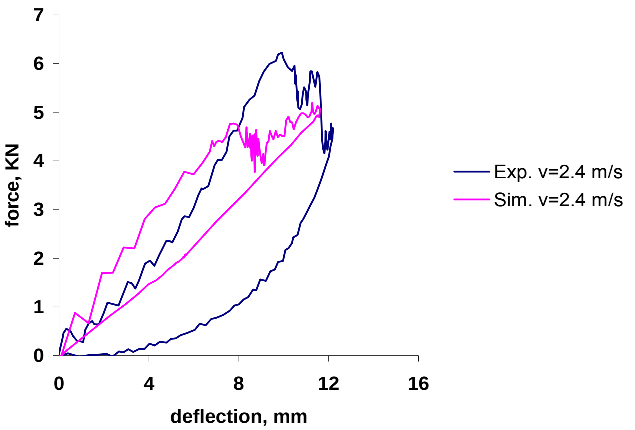


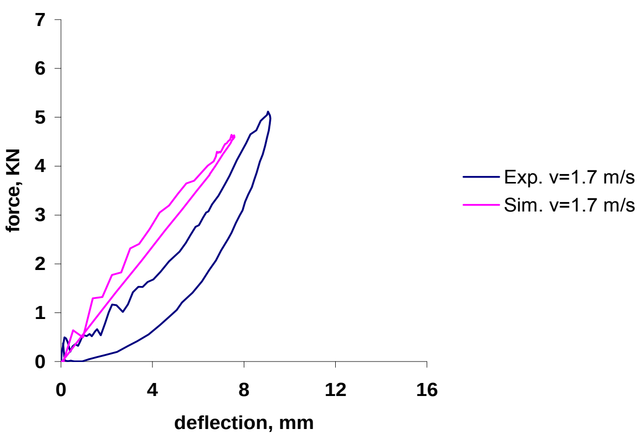
Fig. 5 - Force-deflection curves for [0/90/0] laminate for different impact velocities.



(a)



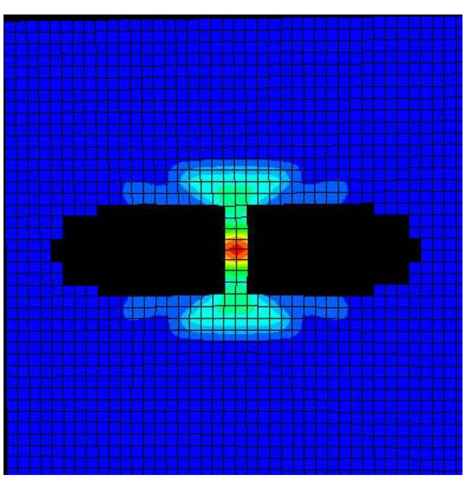
(b)



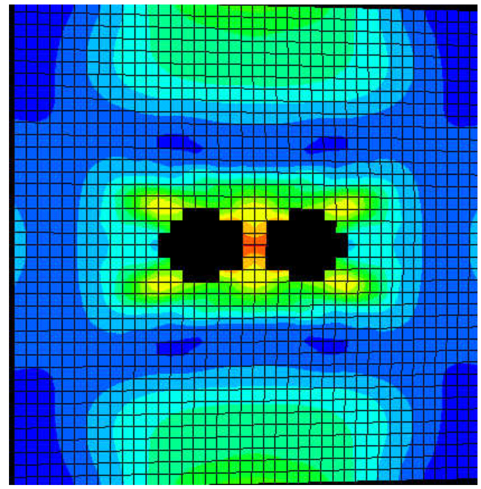
(c)

Fig. 6- Comparison of force-deflection curves at three different impact velocities.

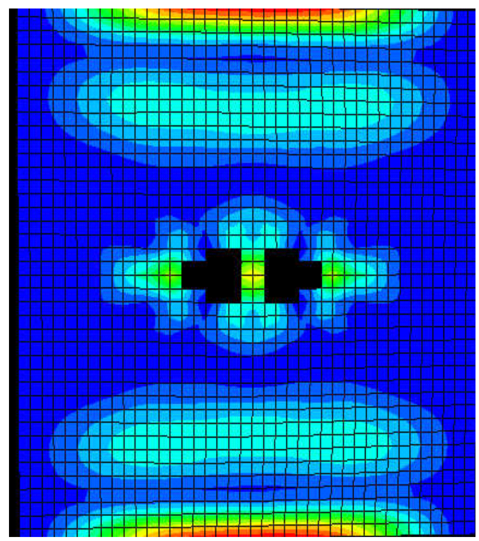
The delamination shape obtained from the simulations for the lower interface of the [0/90/0] at three different impact velocities, 3.6 m/s, 2.4 m/s and 1.7m/s are shown in the Fig. 7(a), 7(b) and 7(c), respectively.



(a) v=3.6 m/s



(b) v=2.4 m/s



(c) v=1.7 m/s

Fig. 7 - Delamination shape at the bottom interface of [0/90/0] laminate at three different impact velocities.

The delamination area vs. Impact energy from both experiments and the simulations are shown in Fig. 8. Clearly the simulations under predicts the delamination area. From Fig. 7 and 8 it can be concluded that the delamination model is only able to capture delamination damage qualitatively but is not able to match the experimental results quantitatively.

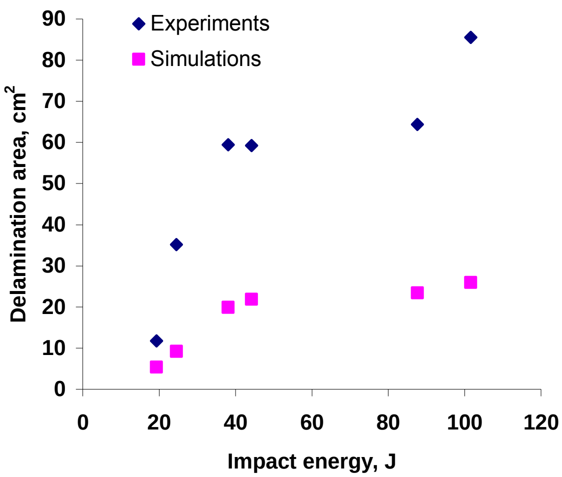


Fig. 8 - Delamination area at different impact energies.

Fig. 9 shows the absorbed energy vs. the impact energy from both experimental and simulation results. The impact energy is based on the kinetic energy of the impactor while the absorbed energy is obtained from the integration of the force-deflection curves.

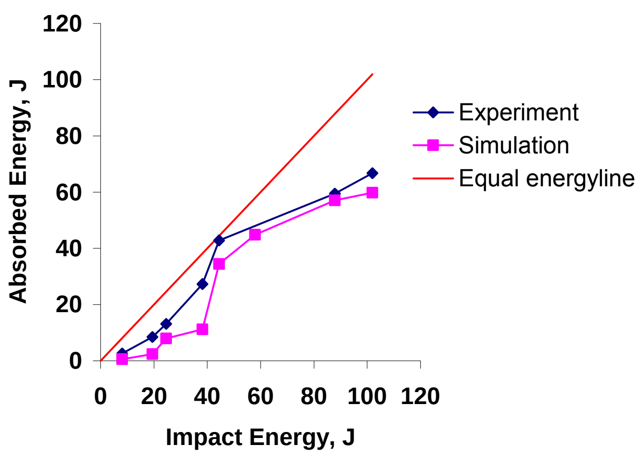


Fig. 9 - Absorbed energy vs. impact energy

To understand the effect of delamination damage on the response of composite laminates, two different simulations were performed. Both simulations were performed for an impact velocity of 2.4 m/s. The first simulation allowed only in-plane damage without delamination damage. The second simulation included both in-plane damage and delamination damage. The force-deflection curves from the two simulations are shown in Fig. 10. The energy absorbed by the composite laminate for the two cases were 6.41 J and 9.91 J. The force-deflection curves and the energy absorbed clearly indicate the role of delamination damage. Delamination caused an increase of about 49 % in the energy absorption.

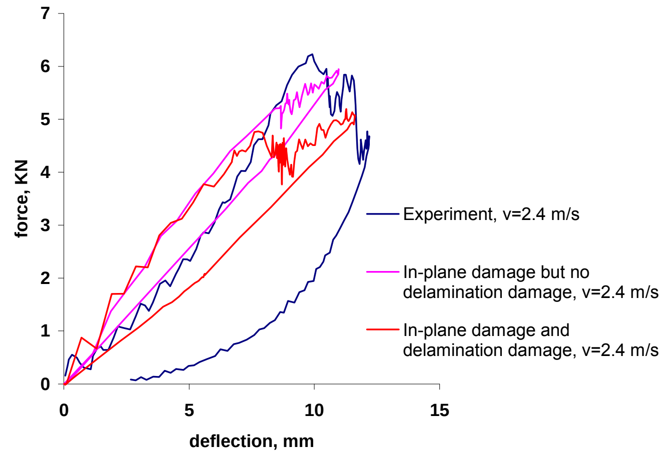


Fig. 10 - Force-deflection curve for [0/90/0] laminate with and without delamination damage.

When finite element simulations were performed for composite laminates with a stacking sequence of [0/30/0] and [0/45/0], there were severe convergence problems. Simulations with only in-plane damage and with only out-of-plane damage converged but simulations including both damage models did not converge.

Fig. 11 compares the force-deflection curves of [0/90/0] with and without delamination, [0/45/0] with out delamination and [0/30/0] without delamination. All the simulations were performed at an impact velocity of 2.4 m/s. It can be noted that for a hypothetical case of only in-plane damage and no delamination damage, the [0/90/0] laminate was the most rigid one followed by [0/45/0] and then by [0/30/0] which had maximum in-plane damage. The energy absorption was 6.41 J, 8.23 J and 9.59 J respectively. This was due to the fact that in a [0/90/0], the 90-lamina arrested the cracks in the 0 lamina due to in-plane damage to propagate.

However, it may be noted that a [0/90/0] laminate has the maximum mismatch of material properties through the laminate thickness and thus would have the maximum delamination. From the earlier discussions, it was known that delamination degraded the laminate drastically. Thus, it was expected that [0/90/0] composite laminate would perform the worst for a real case with where both material in-plane damage and delamination damage took place. The results shown in Fig. 11 seem to point into this direction. Unfortunately, the simulations for [0/45/0] and [0/30/0] composite laminates, did not converge thus making the comparison with [0/90/0] impossible.

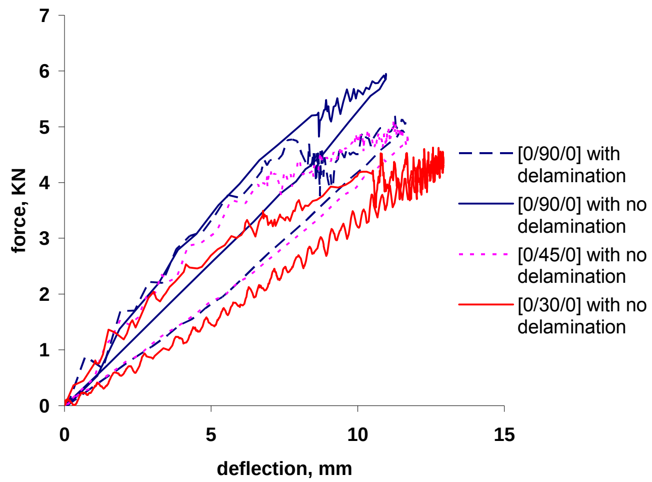


Fig. 11 - Force-deflection curves.

CONCLUSIONS

A finite element model was presented to study the damage process of composite laminates. The commercial finite element code ABAQUS-Standard was used for the computational studies. The finite element model used two-dimensional shell elements, thus making the model computationally efficient. A homogenized continuum model was used to characterize the composite laminates. Material constitutive relations were formulated into a user subroutine UMAT in ABAQUS for in-plane analysis. The failure criteria proposed by Hashin were used. A brittle damage model was used for the post- failure behavior of the matrix while a progressive damage model was used for the fiber damage. For the finite element simulation of the delamination damage, the multi-point constraint (MPC) model was used along with shell elements to represent composite laminate. A user subroutine MPC was written to impose the displacement continuity conditions at the interface. The nodal forces were used to calculate the interlaminar stresses and a stress based failure criterion was used for the delamination damage. Once the delamination was detected, the displacement continuity conditions were removed in the MPC model The delamination model was used to analyze the [0/90] composite beam and the results were compared with the elasticity analysis. The delamination shape and size were validated with different composite layups. A typical penny shaped delamination with the major axis along the direction of the fiber direction of the bottom lamina was obtained but the size predicted from the present model had an error of about 50 % from the experimental results. Two damage models, material in-plane damage model and the out-of-plane damage model for delamination, were used to investigate the response of [0/90/0] composite laminate subjected to impact loading at different impact velocities. The results from the simulations were compared with those from the experiments. The simulation results were in good agreement with the experiments.

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